**Olympiad**
Time: 1.30 Hour
**Stat-Math Academic Cell**

1. Let $a, b, c, d$ and $e$ be the positive integers such that $abcde = a + b + c + d + e$, where $abcde$ denotes the product of $a, b, c, d$, and $e$. Find the maximum possible value of $max\{a, b, c, d, e\}$. [Answer: 5]

2. For any sequence of real numbers $A = \{a_n \mid n \in \mathbb{N} \setminus \{0\}\}$,
define $\Delta A = \{a_{n+1} - a_n \mid n \in \mathbb{N} \setminus \{0\}\}$ . Supoose all the term of the sequence $\Delta(\Delta A)$ are 1, and $a_{19} = a_{92} = 0$. Find the value of $a_1$. [Answer: 819]

3. Let $\lfloor x \rfloor$ denote the largest integer less than or equal to $x$. Find the value of

$$\lfloor \tfrac{1}{4} \rfloor + \lfloor \tfrac{1}{4} + \tfrac{1}{200} \rfloor + \lfloor \tfrac{1}{4} + \tfrac{2}{200} \rfloor + \ldots + \lfloor \tfrac{1}{4} + \tfrac{199}{200} \rfloor$$

[Answer: 50]

4. Find the number of integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 48$ with the conditions such that $x_1 > 5$, $x_2 > 6$, $x_3 > 7$, and $x_4 > 8$. [Answer: 1330]

5. Find the number of real solutions of the equation

$$\sqrt{x^2 + 4x + 4} = x^2 + 3x - 6$$

[Answer: 2]

6. Find the possible number of pairs $(x, y)$, where $x$ and $y$ both are real numbers and which satisfy the system of equations: [Answer: 2]

$$x^3 + y^3 = 1$$
$$x^4 + y^4 = 1$$

7. Find the number of positive integer $x$ which satisfy the inequality

$$\frac{x-1}{x-2} \leq \frac{x-2}{x-1}$$

[Answer: 0]

8. Evaluate the largest integer less than or equal to the number $s$, where

$$s = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots \frac{1}{\sqrt{10000}}$$

[Answer: 197]

9. Find the value of $n$ for which

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{2ab}{a+b}$$

where $a \neq b$. [Answer: -1]

10. Let $X = \{n \in \mathbb{N} \mid n \geq 8\}$. Let $f : X \to \mathbb{R}$ such that $f(x+y) = f(xy)$, for all $x \geq 4, y \geq 4$ and $f(8) = 9$. Determine $f(9)$. [Answer: 9]

11. If 3 numbers are selected from the first 15 natural numbers (not including 0), then what is the probability that the numbers are in A.P ? Give answer up to 3 correct decimal places. [Answer: 0.107 (Correct range: 0.105-0.109)]

12. Find the number of natural numbers $x > 3$ such that $x - 3$ divides $x^3 - 3$. [Answer: 8]

13. Find the number of pairs $(x, y)$, where $x$ and $y$ both are natural numbers (not including 0) which satisy the equation $2^x + 1 = y^2$. [Answer: 1]

14. Let $a$ and $b$ be two consecutive integers selected from the first 1000 natural numbers, not including 0. What is the probability of $\sqrt{a^2 + b^2 + a^2b^2}$ being an odd positive integer? Give answer up to 3 correct decimal places. [Answer: 1.000]

15. Triangle ABC has a right angle at C. The internal bisectors of angles BAC and ABC meet BC and CA at P and Q, respectively. The points M and N are the feet of the perpendiculars from P and Q to AB. Find angle MCN (in degrees).
(Enter the numerical value as your answer. e.g. Suppose your answer is $60^\circ$, then enter 60 as your answer.) [Answer: 45]

16. Let $S = \{(x, y, z) \mid x, y, z$ are non-zero natural numbers satisfying $xy + yz + zx = 2 + xyz\}$. Find the cardinality of the set $S$. [Answer: 7]

17. Let $a, b, c$ be non-zero real numbers such that

$$a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$$

What is the value of $|abc|$? [Answer: 1]

18. Find the least positive integer $m$ such that for all positive integers $n$ we have

$$\binom{2n}{n}^{\frac{1}{n}} < m$$

[Answer: 4]

19. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \frac{2}{4^x+2}$. Find the value of

$$f\left(\frac{1}{2023}\right) + f\left(\frac{2022}{2023}\right)$$

. [Answer: 1]

20. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that

    i) $f(x) \leq x$ for all $x \in \mathbb{R}$.

    ii) $f(x+y) \leq f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

    Determine $f(2022)$. [Answer: 2022]